International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics

1981 - First International Conference on Recent Advances in Geotechnical Earthquake Engineering & Soil Dynamics

27 Apr 1981, 2:00 pm - 5:00 pm

Interface Element for Propagation in Layered Media

V. R. Gadhinglajkar
College of Engineering, Amravati, India

R. M. Belkune
Indian Institute of Technology, Bombay, India

C. K. Ramesh
Indian Institute of Technology, Bombay, India

Proceedings: First International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 26 - May 3, 1981, St. Louis, Missouri

Interface Element for Propagation in Layered Media

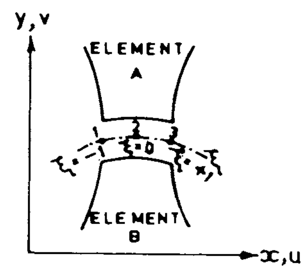
V. R. Gadhinglajkar, Assistant Professor
Applied Mechanics, College of Engineering, Amravati, India

R. M. Belkune, Assistant Professor
C. K. Ramesh, Professor
Civil Engineering Department, Indian Institute of Technology, Bombay, India

SYNOPSIS An interface element which accounts for the relative displacement between two layers through its stiffness properties and loss of energy through its damping properties has been developed. Its behaviour under static, free vibration, transient load and wave propagation in layered media has been studied.

INTRODUCTION

A seismic wave travelling in layered media encounters interfaces at which energy may partly be transmitted through elastic bonding while some of the energy gets dissipated in the relative motion between the layers. The usual practice is to treat interface as perfectly welded/free, which represents extremities of the problem between which the actual situation may lie and has to be provided for. The loss of energy at the interface has also to be considered. The problem of relative displacements between two layers has been tackled by Goodman et al (1968) and Schafer (1975) by using bond (for interface) element.



$N_i = \frac{1}{2}\xi\xi_i(1+\xi\xi_i)$ FOR $i = 1,3$

$N_i = (1-\xi^2)$ FOR $i = 2$

SHAPE FUNCTIONS

FIG. 1 INTERFACE ELEMENT FOR PLANE STRESS/STRAIN

FORMULATION

In the interface element the relative displacement between two layers which is a measure of strain, is accounted for through its stiffness properties and loss of energy through its damping properties (Gadhinglajkar, 1980).

For the plane stress/strain case, Fig. 1 shows an isoparametric interface element connecting two 8-noded isoparametric elements. The relative displacements Δu and Δv at a point are given by

$$\Delta u = u^A - u^B = \sum_{i=1}^{3} N_i\,(u_i^A - u_i^B) \qquad (1)$$

$$= \sum_{i=1}^{3} N_i\,\Delta u_i \qquad (2)$$

Similarly $$\Delta v = \sum_{i=1}^{3} N_i\,\Delta v_i \qquad (3)$$

The strains $\underset{\sim}{\varepsilon}$ (curly line below a letter indicates a column matrix) are given by

$$\underset{\sim}{\varepsilon} = \begin{Bmatrix} \Delta u \\ \Delta v \end{Bmatrix} = \underline{B}\,\underset{\sim}{\Delta} \qquad (4)$$

where $\underset{\sim}{\Delta}$ is the nodal displacement vector. (Line below a letter indicates rectangular matrix). The strains in global directions are transformed to local directions using usual transformation matrix $\underline{T}$.

$$\underset{\sim}{\varepsilon}' = \begin{Bmatrix} \Delta u' \\ \Delta v' \end{Bmatrix} = \underline{T} \begin{Bmatrix} \Delta u \\ \Delta v \end{Bmatrix} \qquad (5)$$

Noting that ξ and η are orthogonal, it can be shown that

$$ds = |J|\,d\xi \qquad (6)$$

and

$$\underline{T} = \frac{1}{|J|}\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ -\frac{\partial y}{\partial \xi} & \frac{\partial x}{\partial \xi} \end{bmatrix} \qquad (7)$$

where J is the Jacobian matrix.
Using Eqns. (4) and (5) one gets

$$\underset{\sim}{\varepsilon}' = \underline{T}\,\underline{B}\,\underset{\sim}{\Delta} \qquad (8)$$

The stress-strain relationship is given by

$$\underset{\sim}{\sigma} = \begin{Bmatrix} \sigma_s' \\ \sigma_n' \end{Bmatrix} = \begin{bmatrix} k_s & 0 \\ 0 & k_n \end{bmatrix} \begin{Bmatrix} \Delta u' \\ \Delta v' \end{Bmatrix} = \underline{D}\,\underset{\sim}{\varepsilon}' \qquad (9)$$

where k_s and k_n are tangential and normal stiffness parameters. The element stiffness matrix $\underline{k}$, now, is obtained as

$$\underline{k} = \int_{-1}^{1} \underline{B}^T \underline{T}^T \underline{D} \underline{T} \underline{B} \, |J| d\xi \tag{10}$$

(Superscript T indicates transpose of a matrix.) $\underline{k}$ is integrated numerically using 3-point Gauss quadrature rule.

The damping force $\underset{\sim}{f}$ is assumed as

$$\underset{\sim}{f} = \begin{bmatrix} c_s & 0 \\ 0 & c_n \end{bmatrix} \begin{Bmatrix} \Delta \dot{u}' \\ \Delta \dot{v}' \end{Bmatrix} = \underline{\mu} \, \dot{\underset{\sim}{\varepsilon}}' \tag{11}$$

where c_s and c_n are the tangential and normal damping parameters. (Dot on top indicates differentiation with respect to time.) Considering the work lost in damping over the element and the work done by the nodal damping forces, the element damping matrix $\underline{c}$ is given by

$$\underline{c} = \int_{-1}^{1} \underline{B}^T \underline{T}^T \underline{\mu} \, \underline{T} \underline{B} \, |J| d\xi \tag{12}$$

For the axisymmetric case the corresponding element matrices are

$$\underline{k} = 2\pi \int_{-1}^{1} \underline{B}^T \underline{T}^T \underline{D} \underline{T} \underline{B} \, r \, |J| d\xi \tag{13}$$

$$\underline{c} = 2\pi \int_{-1}^{1} \underline{B}^T \underline{T}^T \underline{\mu} \, \underline{T} \underline{B} \, r \, |J| d\xi \tag{14}$$

where r is the radius.

Following the above steps and with appropriate modifications to displacement functions an infinite interface element is derived, Gadhinglajkar (1980).

NUMERICAL STUDIES

Numerical studies for stresses and deflections on cantilever with interface element subjected to static tip loads have been made. Table 1 gives influence of k_s and k_n on free vibration of the cantilever shown in Fig. 2.

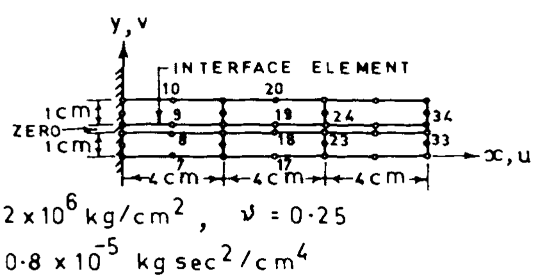


FIG. 2 CANTILEVER WITH INTERFACE ELEMENT

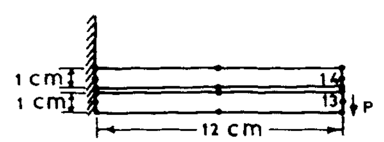


FIG. 3 CANTILEVER WITH INTERFACE ELEMENT

In Table 2 are presented results of inclu̇ence of c_s on displacement of the cantilever, Fig. 3. Comparison of displacements with $c_s = 0.657 \times 10^{-1}$ kg-sec/cm, $c_s = 6.57$ kg-sec/cm has been made, with those obtained for material damping assumed proportional to mass and stiffness, corresponding to the extreme frequencies of 0.307×10^4 rad/sec and 0.992×10^7 rad/sec of the cantilever, Fig. 3, and 5% of the critical damping.

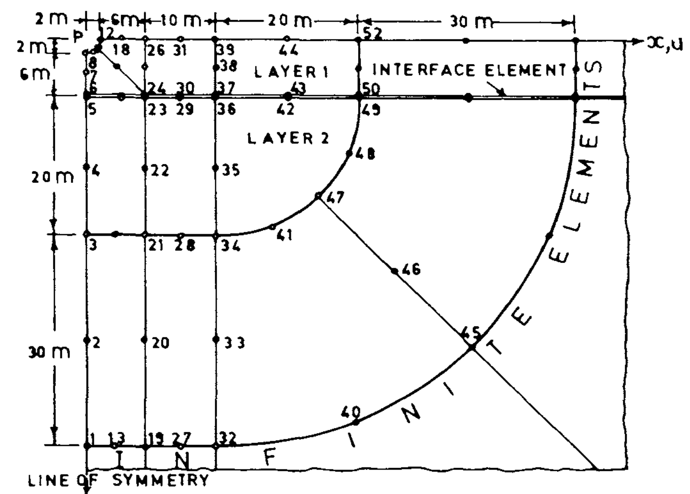
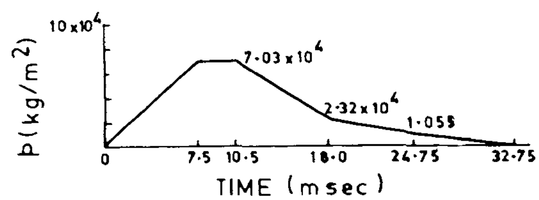


FIG. 4 FINITE ELEMENT MESH FOR LAYERED MEDIA-SOURCE ON SURFACE



$E_1 = 0{\cdot}8 \times 10^9$ kg/m^2 , $E_2 = 3 \times 10^9$ kg/m^2

$\rho = 228{\cdot}6$ kg sec^2/m^4 , $\nu = 0{\cdot}25$

FIG. 5 LOAD PULSE FOR LAYERED MEDIA

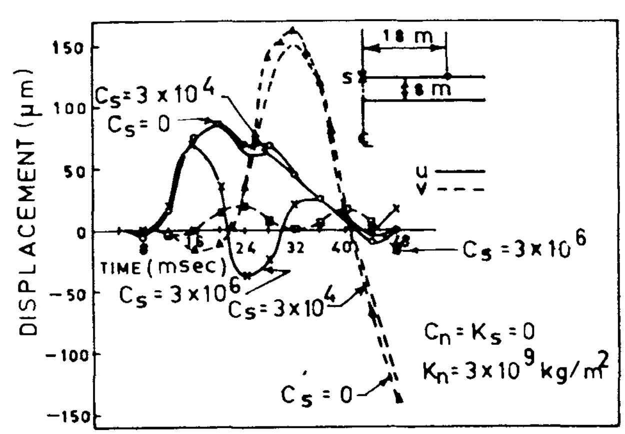


FIG. 6 EFFECT OF C_s ON DISPLACEMENT AT 18 m ALONG FREE SURFACE

TABLE I. Effect of k_s and k_n on Frequency of Cantilever with Interface Element
Frequency in rad/sec.

Type / Mode →	1	2	3	4	5	Last
i) Solid (without joint element)	0.681×10^4 (0.7042×10^4)*	0.37×10^5	0.654×10^5	0.886×10^5	0.181×10^6	0.346×10^7 (48th)
ii) With joint element $k_s = k_n = 10^6$	0.574×10^4	0.302×10^5	0.596×10^5	0.695×10^5	0.159×10^6	0.295×10^7 (60th)
iii) With joint element $k_s = k_n = 10^8$	0.58×10^4	0.317×10^5	0.597×10^5	0.756×10^5	0.173×10^6	0.809×10^7 (60th)

* Fundamental Frequency for solid cantilever (Warburten, 1976)

TABLE II. Influence of c_s on displacements of Cantilever with Interface Element
$t = 0.3\ \mu sec$ Displacements (cm) after 80 time steps

	c_s	Node 13 u	Node 13 v	Node 14 u	Node 14 v
1	$c_s = 0.0$	0.2139×10^{-4}	-0.1618×10^{-3}	-0.213×10^{-3}	-0.1618×10^{-3}
2	$c_s = 0.657 \times 10^{-4}$	0.2139×10^{-4}	-0.1618×10^{-3}	-0.213×10^{-4}	-0.1618×10^{-3}
3	$c_s = 0.657 \times 10^{-2}$	0.2121×10^{-4}	-0.1618×10^{-3}	-0.2112×10^{-4}	-0.1618×10^{-4}
4	$c_s = 0.657 \times 10^{-1}$	0.1966×10^{-4}	-0.1614×10^{-3}	-0.1958×10^{-4}	-0.1614×10^{-4}
5	$c_s = 0.657 \times 10^{1}$	0.1923×10^{-5}	-0.1560×10^{-3}	-0.1836×10^{-5}	-0.156×10^{-3}
6	$c_s = 0.657 \times 10^{3}$	0.6379×10^{-7}	-0.155×10^{-3}	0.2389×10^{-7}	-0.155×10^{-3}

$k_s = 0,\ k_n = 10^8,\ c_n = 0$

TABLE III. Comparison of Displacements for Different Values of c_s and α and β for Cantilever with Interface Element. $t = 0.3\ \mu sec$; Displacements in cm

Time-step	Node 13 u (a)	u (b)	u (c)	v (a)	v (b)	v (c)	Node 14 u (a)	u (b)	u (c)	v (a)	v (b)	v (c)
20	0.2881×10^{-5}	0.6877×10^{-6}	0.2982×10^{-5}	-0.232×10^{-4}	-0.2305×10^{-4}	-0.2318×10^{-4}	-0.2804×10^{-5}	-0.6110×10^{-6}	-0.2905×10^{-5}	-0.2286×10^{-4}	-0.2271×10^{-4}	-0.2284×10^{-4}
40	0.1478×10^{-4}	0.3264×10^{-5}	0.1532×10^{-4}	-0.8275×10^{-4}	-0.8118×10^{-4}	-0.8270×10^{-4}	-0.1471×10^{-4}	-0.3196×10^{-5}	-0.1525×10^{-4}	-0.8274×10^{-4}	-0.8117×10^{-4}	-0.8270×10^{-4}
60	0.2325×10^{-4}	0.4029×10^{-5}	0.2431×10^{-4}	-0.1403×10^{-3}	-0.136×10^{-3}	-0.1403×10^{-3}	-0.2348×10^{-4}	-0.4259×10^{-5}	-0.2454×10^{-4}	-0.1406×10^{-3}	-0.1363×10^{-3}	-0.1406×10^{-3}
80	0.1966×10^{-4}	0.1923×10^{-5}	0.2132×10^{-4}	-0.1614×10^{-3}	-0.156×10^{-3}	-0.1613×10^{-3}	-0.1958×10^{-4}	-0.1836×10^{-5}	-0.2124×10^{-4}	-0.1614×10^{-3}	-0.156×10^{-3}	-0.1613×10^{-3}

(a) $c_s = 0.657 \times 10^{-1}$, $c_n = 0 = \alpha = \beta$. (b) $c_s = 0.657 \times 10^{-1}$, $c_n = 0 = \alpha = \beta$. (c) $c_s = 0 = c_n$, $\alpha = 307$, $\beta = 1.01 \times 10^{-8}$

Numerical studies have also been made to study the effect of consistent and lumped damping. Lumped damping matrix is obtained by scaling the diagonal terms of the latter so as to preserve the total element damping. Figs. 4 and 5 show finite element mesh and load pulse applied in small cavity for the wave propagation in two layered media. Effect of c_s on displacements at 18 m along the free surface has been shown in Fig. 6. Influence of c_s and k_s on the particle motion diagrams at nodes 36 and 37 has been shown in Fig. 7.

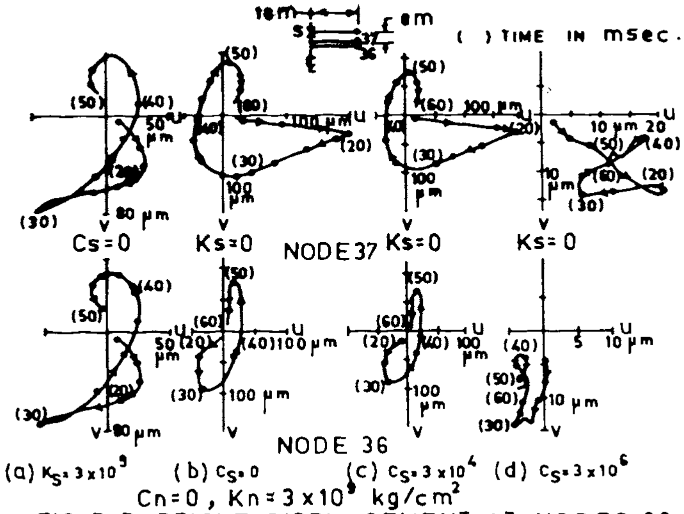


FIG.7 PARTICLE DISPLACEMENT AT NODES 36 AND 37-EFFECT OF Ks AND Cs

DISCUSSIONS AND CONCLUSIONS

To highlight the applicability of the interface element a simple cantilever has been studied. Influence of k_s and k_n on vertical tip deflection and fibre stresses under static load is studied. Results for the extreme cases of bonded and perfectly smooth cases showed good agreement with the alternative solutions. It is also observed that the modelling of cantilever by 3 elements across the span (total 9 elements) and 1 element across the span (total 3 elements) does not make appreciable difference in tip deflection.

It is seen from the results of free vibration of the cantilever, Table I, that for $k_s = k_n = 10^8$ kg/cm^2 the frequencies are higher compared to those for $k_s = k_n = 10^6$ kg/cm^2; the difference, being small in the initial modes, gradually increases in the higher modes. This is so because with the higher stiffness values, the higher modes will get more and more separated from those obtained with the lower stiffness values.

The values of k_s and k_n in the range of the value of E (the modulus of elasticity) correspond to perfect bonding through interface.

In the case of cantilever subjected to sinusoidal load, for want of adequate data the values of c_s and c_n are assumed to be related to critical damping for the media material. It is seen from Table II that significant change in vertical displacement occurs for c_s varied from 0.657×10^{-1} kg-sec/cm to 0.657×10^{1} kg-sec/cm. The vertical displacements for the three cases shown in Table III, do not differ appreciably, the values in (a) and (c) being practically the same; while the horizontal displacements in (a) and (c) are closer and those for (b) differ considerably. The value of c_s in (a) may, therefore, be considered equivalent to material damping in (c) in the range of time considered.

Comparison of lateral tip deflection of the cantilever for the cases of consistent and lumped damping showed insignificant difference in the results.

In wave propagation in two layered media, Fig. 4, to study the influence of c_s on displacements, initial value of c_s has been taken as 1.971 kg-sec/m on the presumption that the effect of c_s will be apparent for $\frac{c_s}{k_n} = \frac{0.657 \times 10^{-1}}{10^8}$ as has been the case with the cantilever. However no impact of c_s on displacements has been felt. Hence the values of c_s tried have been 3, 30, 3×10^2, 3×10^3, 3×10^6 kg-sec/m. Upto 3×10^3 kg-sec/m no change in displacements has been observed; at 3×10^4 kg-sec/m the change in response becomes apparent and at 3×10^6 kg-sec/m the response is dampened almost entirely and loses its oscillatory character significantly. Fig. 6 shows that with $c_s = 3 \times 10^4$ kg-sec/m displacements get reduced marginally compared to those with $c_s = 0$, indicating that a small amount of energy is absorbed at the interface.

Influence of k_s on particle motion diagram, has been shown in Fig. 7. Because of nearly perfect bonding at interface due to k_s, the displacements on both sides of the interface show identical pattern. Comparison of influence of c_s on the particle motion at a typical pair of nodes on interface, Fig. 7, shows that for $c_s = 0$ and $c_s = 3 \times 10^4$ kg-sec/m the particle motion diagrams at the nodes are similar. It is also observed that with $c_s = 3 \times 10^4$ kg-sec/m there is reduction in horizontal and vertical displacements. However reduction in horizontal displacements, as is expected, is more than that in the vertical component, because c_s is horizontal in this case. For $c_s = 3 \times 10^6$ kg-sec/m the vertical displacements are not affected appreciably but because of the high value of c_s, the horizontal displacements are reduced and are also out of phase.

ACKNOWLEDGEMENT

The authors are grateful to the Council of Scientific and Industrial Research, New Delhi and the Centre of Studies in Resources Engineering, Indian Institute of Technology, Bombay for the financial support for carrying out these studies.

REFERENCES

Gadhinglajkar, V.R. (1980), " Finite Element Formulation for Shock Wave Propagation" Thesis submitted for the degree of Doctor of Philosophy, Dept. of Civil Engg., I.I.T. Bombay, India.

Goodman, R.F. et al (1968), " A Model for Mechanics of Jointed Rock" , Proc. ASCE 94, SM 3, pp 637-659.

Schäfer, H. (1975), " A Contribution to the Solution of Contact Problems with the Aid of Bond Elements" , Comp. Methods in Appl. Mech. and Engg., vol. 6, pp 335-354.

Warburten, G.B. (1976), " Dynamical Behaviour of Structures" , Pergamon Press, New York.